Reg. No. :

Question Paper Code : 52761

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017

First Semester

Civil Engineering

MA 2111 – MATHEMATICS – I

(Common to All Branches)

(Regulations 2008)

Time : Three Hours — Maximum : 100 Marks

Answer ALL questions.

PART – A (10×2=20 Marks)

1. Two eigen values of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are equal to 1 each. Find the eigen values of A^{-1}.

2. State Cayley – Hamilton theorem.

3. Find the equation of the sphere whose centre is (2, –3, 4) and radius 5.

4. State a quadric cone.

5. Find the radius of curvature at any point (x, y) on $y = c \log \sec \frac{x}{c}$.

6. Define circle of Curvature.

7. If $u = \frac{y}{z} + \frac{z}{x}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.

8. If $x = u + uv$, $y = v + uv$, find $\frac{\partial(x,y)}{\partial(u,v)}$.

9. Evaluate $\int_0^1 \int_1^2 \left(x^2 + xy\right) dydx$

10. Sketch the region for $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} f(x,y)\,dydx$.

PART – B (5×16=80 Marks)

11. a) i) Find the canonical form of the quadratic form (10)

$2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$.

ii) Prove that the eigenvalues of a real symmetric matrix are real numbers. (6)

(OR)

b) i) Find the eigen values and eigen vectors of $\begin{pmatrix} 6 & -6 & 5 \\ 14 & -13 & 10 \\ 7 & -6 & 4 \end{pmatrix}$ (8)

ii) Verify Cayley-Hamilton theorem for the matrix $\begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$ (8)

12. a) i) Find the equation of the sphere which passes through the circle $x^2 + y^2 + z^2 = 5$ and $x + 2y + 3z = 3$ and touches the plane $4x + 3y = 15$. **(8)**

ii) Find the equation of the cone whose vertex is (3, 1, 2) and base curve $2x^2 + 3y^2 = 1, z = 1$. **(8)**

(OR)

b) Find the equation of the right circular cylinder described on the circle through the points (a, 0, 0), (0, a, 0), (0, 0, a) a guiding curve. **(16)**

13. a) i) Find the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$. **(8)**

ii) Find the equation of the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ **(8)**

(OR)

b) i) Considering the evolute as the envelope of the normals, find the evolute of the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. **(10)**

ii) Find the centre of curvature of $y = x^2$ at the origin. **(6)**

14. a) i) If u is a homogeneous function of degree n in x and y, show that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = n(n-1)u$. **(8)**

ii) If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, then find the value of $x^2u_x + y^2u_y + z^2u_z$. **(8)**

(OR)

b) i) Find the maximum and minimum of

$\sin x \sin y \sin(x+y),\ 0 < x < \pi,\ 0 < y < \pi$. **(10)**

ii) Expand the function sin xy in powers of $x-1$ and $y-\frac{\pi}{2}$ up to second degree term. **(6)**

15. a) Change the order of integration in $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\,dx\,dy$ and hence evaluate the same. **(16)**

(OR)

b) i) Evaluate $\iiint_V \frac{dz\,dy\,dx}{(x+y+z+1)^3}$ over the region of integration bounded by the planes $x=0,\ y=0,\ z=0,\ x+y+z=1$. **(8)**

ii) Find the volume of the solid surrounded by the surface

$\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}+\left(\frac{z}{c}\right)^{\frac{2}{3}}=1$. **(8)**